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The Role of Applied Problems from the School Mathematics Course for the Learning Purposes

ABSTRACT

of the PhD Dissertation

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INTRODUCTION

The mind is not only in the knowledge, but also the ability to apply it.
Aristotle

Relevance of the problem

„ We have a problem when there is a contradiction between the new requirements and the existing reality.“ (Бижков & Краевски, p. 155)

Problems of our educational system are the low functional literacy of students, their difficulties to cope with tasks requiring combining multiple knowledge and approaches. A **problem** is the easy refusal to solve problems set in implicit form. It is a **problem** that students are expected to develop new competencies – creativity, critical thinking, problem-solving, dealing with information – but their preparation does not meet these requirements. A **problem** is the existence of few and isolated applied tasks in the high school mathematics course in our country. "For students to be able to apply mathematical knowledge in non-mathematical contexts, training in the spirit of mathematical modelling is necessary. It is important that students not only know some mathematical applications and standard models, but also have experience in the process of active modelling." (Събева-Колева, 2010)

According to the mathematics curricula of the Ministry of Education and Science (MES) in grades 9, 10, 11 and 12, learning "is aimed at mastering the basic knowledge, skills and attitudes related to the achievement of the learning outcome requirements in the subject of mathematics and to the development of the student's key competencies." (МОН, 2017).

The European reference framework specifies 8 competences:

- language literacy, multilingual competence,
- mathematical competence and competence in the field of exact sciences, technology, and engineering,
- digital competence,
- personal competence, social competence,
- competence to acquire learning skills,
- civil competence,
- entrepreneurial competence
- cultural awareness and performance competence.
  (Съвет на Европейския съюз, 2018)

The European Digital Competence Framework (DigComp, 2023) lists five key competences: information literacy; communication and collaboration; digital content creation; safety; problem solving skills.

The school mathematics course plays a key role in the development of the listed

competencies. On the other hand, applied tasks are important for the development of the following the **21st century skills**:

- reading literacy,
- mathematical literacy,
- problem-solving,
- study skills,
- critical thinking,
- digital literacy,
- creativity,
- handling the information provided,
- self-direction and self-evaluation.

At the beginning of the 19th century, the German mathematician Felix Klein in his book "Elementarmathematik vom Höheren Standpunkte aus" ("Elementary mathematics from the point of view of higher mathematics") (Klein, 1908) addressed the problem of the lack of applied problems as part of the school mathematics course. He recommended that teachers "… take many examples from life for your own teaching" (Klein, 1908). Two centuries later, the concept, observations and recommendations of Felix Klein continue to sound relevant.

Topic

The topic of the dissertation is the role of applied tasks in the secondary school mathematics course for the purpose of education – mental and intellectual development of students. The role of these tasks is multifaceted and complex. On the one hand, they are a tool for developing learners' cognitive and metacognitive skills, analytical thinking, creativity and functional literacy. On the other hand, they are a stimulus for motivation and engagement. They are an intellectual and cognitive challenge for teachers and students.

Object of the study

The object of the study is the use and application of knowledge of functions in solving practical problems in secondary school mathematics education. "Pupils perceive the problem as a learning necessity. The teacher's role is to turn this necessity into an interesting challenge... He must coordinate and correct the students' problem-solving behaviour... Mathematics teaching itself has to be problem oriented". (Тонова, 2012)

Subject of the study

The subject of the study is the importance of applied tasks for the development of cognitive and metacognitive skills and competences of students in the first and second years of secondary school.

Aim of the study

The study of mathematics at school aims not only to teach subject knowledge but also to develop logical thinking, analytical reading and reasoned argument.

The aim of this dissertation is an exploratory cross-section of applied tasks with regard to their role in developing students' mathematical thinking, skills and competencies.

This dissertation examines how competencies "come alive" through applied tasks. The literature studied, best practices from foreign education systems reviewed, and a review of projects developed in this country provide concrete examples of tasks that illustrate the importance of mathematics in problem solving of various kinds.

1. **Task**: To examine similar studies considering the methods used, the hypotheses underlying them, and the conclusions drawn.
2. **Task**: To compare, on the one hand, the normative documents, and programmes in force in the Republic of Bulgaria for education in the lower secondary and upper secondary stage and, on the other hand, the EC documents for the development of education, especially those concerning Bulgaria as a member of the European Union.
3. **Task**: To compare the current textbooks in Bulgaria and foreign textbooks by systematizing good practices from other educational systems that can be adapted and enrich the mathematics education.
4. **Task**: To analyse the role of applied tasks as a tool for making interdisciplinary connections. A school course of study is a collection of knowledge and skills in various fields. Tasks combining issues from different fields offer a unifying model of cognition.
5. **Task**: To find out to what extend there is a need and how it is possible to supplement the mathematics curriculum with applied tasks without reducing the theory and routine tasks that are laid down in the current textbooks.
6. **Task**: To describe and explore the axes of consideration of the role applied tasks.
7. **Task**: To describe and argue the skills – cognitive and metacognitive, the 21st century skills and the competences that are developed through the applied tasks.

Hypotheses

According to the Strategic Framework for the Development of Education, Training and Learning in the Republic of Bulgaria (2021 – 2030) "The main goal of teaching is to make learning more attractive and practically oriented by applying new methods and approaches and to build attitudes towards learning through whole life." (МОН, Стратегически документи, 2021)

The modern student pays less and less attention to reading. Habits of fast, superficial, diagonal reading are inapplicable when working with a text-intensive mathematical problem. Only careful reading can separate the important information from the superfluous and highlight the necessary components for building the mathematical

model.

Despite curriculum changes and new textbooks, applied problems remain underrepresented in the upper secondary curriculum, which is not in line with the level of preparation in other European countries. The EC recommends the exchange of good practice and partnership in education without compromising the identity of individual systems. Bulgaria has a tradition of in-depth theoretical mathematical education of which we can be proud. In connection with the new changes in the curriculum and the introduction of a new format for the State Baccalaureate Examination (SBE), inspiration for a variety of tasks should be drawn from the experience and best practices of other countries.

Teachers need to develop the ability to be creative when working with textbook tasks and/or to create appropriate applied tasks according to the learning objectives, the needs of the students and within the learning material. They should also be able to select appropriate tasks to develop students' cognitive skills and to suggest different strategies for solving them. The teacher's role is extended to that of a facilitator, encouraging analytical and critical thinking and creativity. The teacher not only chooses the problems and creates an environment for discussion and the search for an optimal solution, but also must stimulate students to explore, to argue, to look for alternative approaches, to create problems.

1. Hypothesis

Applied tasks are one of the tools for achieving the objectives set out in the strategic framework. They increase interest and motivation by providing examples of the use of specific mathematical knowledge in different areas of life and develop lifelong learning skills.

2. Hypothesis

Working on applied problems largely develops 21st century skills and prepares students for the next stage of their education. Such tasks help to achieve the objectives of mathematics education by developing the identified key competences – critical thinking, analytical reading, problem solving, creativity – to a greater extent. The transition from text to mathematical model creates a link between theory and practice. Creativity is expressed both in the solution of a specific problem and in the ability to create new ones.

3. Hypothesis

Applied problems are more difficult even for students with a good mathematical background. The variety of the problems is important for the development of analytical-heuristic thinking.

4. Hypothesis

Purposeful work on such problems is important for building cognitive and

metacognitive patterns, for training focused reading, and for increasing self-discipline.

5. Hypothesis

In view of globalisation, mobility, and the goal of a European Education Area (EEA) by 2025 (Commission, 2023), it is necessary to supplement the course of study with practical tasks in line with the set curriculum content. This will help to increase the competitiveness and mobility of school leavers (Directorate-General for Education, 2023).

6. Hypothesis

In the context of lifelong learning, teachers also need to develop their skills, learn, seek, and explore new educational technologies and tools.

Research methods

Triangulation refers to the use of multiple methods or data sources in qualitative research to develop a comprehensive understanding of phenomena (Patton, 1999). In the present dissertation, the triangulation is between (1) scientific literature, publications, studies, and relevant statistics; (2) various textbooks and curricula – international and Bulgarian over the years; (3) field research and case-study. The set and used indicators, methods and tools of the study are described in detail in the fifth chapter.

Structure and content

This **introduction** deals with the problem and its significance. It presents the topic, object, and subject of the dissertation. The introduction defines the aims and objectives of the study, states the hypotheses, and describes the research method. It also describes the structure of the dissertation.

The **first chapter** defines the main concepts used in the exposition. Based on a literature review, and regarding the aims of the thesis, the questions used are described – wording, structure, main parts, vocabulary, diversity. The methodology of constructing questions according to their levels of complexity is discussed. An example of the process of selecting tasks is given. Selected criteria for evaluating the importance of the applied tasks in the educational course are given. The applied problems and their solutions are considered from different perspectives. The stages of problem solving, the expected activities of the students, the role of the teacher, the competences addressed, the cognitive and metacognitive skills are analysed. Knowledge and skills, competences and 21st century skills, functional and multiple literacy are described in the context of the given theme.

In the **second chapter**, a review of education and the role of mathematics in the learning process is made, based on a literature review and current digital applications based on artificial intelligence. The new educational paradigm of lifelong learning and its implications for teachers and students are examined. Attention is paid to motivation

through the prism of applied tasks and the importance of creative thinking in school mathematics is traced.

The **third chapter** reviews the attitudes of parents and students towards mathematics preparation at different stages of education. An explanation is sought for the discrepancy between expectations and results. The paradox between interest in mathematics and the actual acquisition of lasting knowledge and skills is examined. As an example, the participation of representatives of the different age groups in the European Kangaroo competition is considered. Through the prism of applied tasks, the development of the solution to a specific task is followed through the different stages of learning, following the spiral model of education. The aim is to analyse the added value of the applied tasks and their role in the development of competences. In view of the problems identified, the shared experience of specialists in the field is sought through a review of scientific publications on the subject.

In the **fourth chapter**, a parallel is drawn with foreign systems from the point of view of applied tasks. Examples of individual tasks are considered, as are approaches to the study of different methodological units. The differences are described and the approaches and tasks necessary for the development of a unified European education are identified. Specific examples of applied tasks at different levels of education are analysed. The place and role of digital technologies in mathematics education is described. Contemporary pedagogical practices such as project-based learning are reviewed. The mathematical essay as a tool in mathematics education is described in detail. A section is devoted to applied tasks according to Bloom's taxonomy in the context of the competency model.

The **fifth chapter** describes the preparation of the field research. It starts with preliminary observations that give direction for the selection of a specific mathematical task, formulating two versions of it – for the control group and for the focus group. The supplementary questionnaire is presented. The choice of methods and instruments is justified. Evaluation criteria and indicators are defined. The mathematical preparation of the participants in the experiment – high school students – is characterised. The process of conducting the experiment twice in two consecutive years is described, as well as the conditions of implementation.

The **sixth chapter** describes the coding of the data collected during the two phases of the study. Several approaches to solving the problems are examined and the students' solution strategies are commented on. A part of the decisions is offered according to the established criteria and indicators. A report on the results of both phases is presented. The survey results and data are analysed in the context of 21st century skills. The qualitative analysis of the results from the point of view of the role of applied tasks in mathematics education provides a basis for discussion and conclusions.

The **seventh chapter** presents five specific examples of applied tasks suitable for the second stage of upper secondary education – higher level. The proposed solutions are considered from the perspective of the competence approach and the development of 21st century skills. Pupils' creativity is seen as a potential source of various applied tasks.

The **conclusion** summarises the results of the research in the field, from the current normative documents, from the reviewed literary sources, from the foreign educational practices. The proposed hypotheses are compared with the conclusions drawn. Possible directions for development of research and analysis of results are outlined.

The dissertation ends with a reference to the author – publications, participation in scientific projects and contributions to the presented development.

1. CHAPTER: BASIC CONCEPTS

The dissertation examines the importance of **applied problems** for the development of skills and competences at **cognitive** and **metacognitive** levels. There are different definitions of task in the literature. The starting point of the presentation is to specify the meaning of each term. On an intuitive level, the terms seem clear, which does not guarantee their full understanding and use. The overview of the scientific literature on the problem serves to describe them unambiguously, so that their use in the dissertation can be complete and clear.

1.1. Problem

- Task, exercise, drill
- Word problem
- Applied task – structure and challenges:
    - the meaning and role of the wording and the vocabulary used,
    - main parts,
    - the structure of the questions by levels of complexity,
    - criteria for the cognitive role of the tasks,
    - processes of creation,
    - ways of achieving diversity,
    - building interdisciplinary.

Applied task with a parallel to fictional concepts:

1. Describe the situation and the people involved. Not all the information is necessary for the decision, but it creates a framework that is close to a real situation. By nature, people love fairy tales, invent stories, tell about circumstances and processes. The applied task has its plot, i.e. it starts with an **introduction**.

2. An action or event that changes a situation. By analogy, this is the connection – a collision, a **conflict**.

3. A problem has arisen – a **climax**.

4. Questions to the decision maker about the situation. Here the problem solver takes on the role of creator and must resolve the conflict – **denouement**.

5. Question – how to solve the problem that has arisen. The final answer gives a conclusion. The Decider has gone from listener to hero in the denouement and is left to write the **epilogue**.

Depending on the maturity of the students, the difficulty can be increased by "reversing" the structure – starting with the problem question and then describing the situation in which the problem arose.

1.2. Solution of a word problem

Problem solving begins with a lack of clear steps and conscious ideas. According to Lester & Kehle, successful problem solving involves the coordination of prior experience, knowledge, familiar representations, inference patterns and intuition to perform cognitive activities, some of which are non-routine. (Lester F. K., 2003) One of the models considered in the thesis is the one shown in Fig. 1. The dashed arrows indicate how the solution found should be compared and interpreted with the other phases. The solver moves between two worlds – that of the problem and that of mathematics. At each stage there is a comparison and a reasonableness check. The number of such comparisons can be an indicator of the complexity of the problem. (Lester F. K., Thoughts About Research On Mathematical Problem- Solving Instruction, 2013)

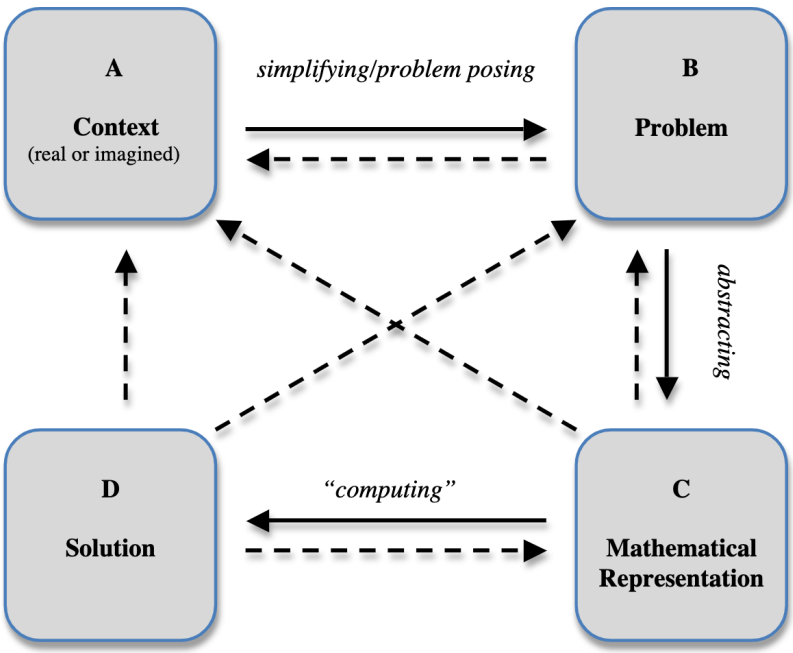

Fig. 1: A model of complex mathematical activity

Table 1provides a more general description of the activities and objectives that should be useful for both the teacher and the individual student. In the original table there are two axes of consideration – activity and purpose. A third axis – 'Cognitive meaning' – which includes cognitive and metacognitive skills and competences, has been added to the problematic aspect of the research.

Table 1: Approach to solving an applied problem.

Activity	Goal	Cognitive meaning
Before solving the task		
Reading the assignment; Discussing words or phrases that are familiar or not completely clear.	The Importance of Focused Reading, Attention to vocabulary	Analytical reading, Concentration, Handling of information provided.
Re-reading until a complete understanding of the formulated task; Insight into the essence of the problem.	Focusing on the important data, Deep understanding of the content.	Critical thinking, Mathematical literacy, Discipline.
During solving		
Observation, Finding out what the progress is in working on the solution.	The teacher determines how well the material has been mastered; The student can revise his approach.	Self-direction, Motivation, Mathematical knowledge, *Problem-solving*
The teacher gives directions, The student considers similar tasks that he has successfully solved, considers different methods and strategies.	Overcoming a blockage that has occurred.	Self-regulation, Mathematical knowledge, *Problem-solving*, Creativity.
Formulating an answer.	Looking back whether the answer made sense.	Critical thinking, Self-assessment.
After solving is complete		
Analysis of the decision.	Discussing different strategies.	Communication, Mathematical literacy.
Parallel with similar tasks.	Discuss the general applicability of the strategies.	Mathematical knowledge, *Problem-solving*, Creativity.
Analysis of the specifics of the task.	How the individual elements of the condition provide guidance for choosing a solution approach.	Analytical reading, Handling of information provided.

As a result of the present study, by combining and extending the strategies described in the literature, following the stages of D. Polya, with attention to the problem under consideration, the dissertation proposes an additional fragmentation of the decision phases.

1.3. Competence and the 21st century skills

The object and subject of the present dissertation is the development of competences and most of the 21st century skills through work on applied tasks. For the purposes of the study, the following parallel has been drawn between them (Table 2).

Table 2: Competences and the 21st century skills

Competencies	**21st century skills**
Ability to use knowledge and skills	Literacy, math, and digital literacy
Taking responsibility for decision making	Critical thinking, *problem-solving*
Degree of independence	Learning, self-direction and self-assessment skills; Handling of information provided
Review and development of own achievements	Critical thinking
Innovation	Creativity

The **first chapter** is a detailed description of the following concepts, which are necessary for the purposes of the thesis:

- knowledge,
- skill,
- cognitive and metacognitive skills,
- functional and multiple literacy.

2. CHAPTER: MATHEMATICS IN EDUCATION

Goal setting is leading in education. The theme and subject of the dissertation follow the understanding of Iv. Tonov "The process of building mathematical thinking goes through mastering different approaches and techniques to math tasks." (Tonov, 2012)

In view of the set goals of the study, Alan Schoenfeld's thesis on the role of mathematics education is followed:

- To enable students to get an idea of the discipline.
- To develop students' understanding of the concepts – not to mechanically apply techniques, but to be able to flexibly handle what they have learned.
- To enable students to explore a wide range of tasks ranging from exercises, applied tasks, analysis, and synthesis tasks. To provide a rich set of approaches and techniques for solving – direct application of the appropriate algorithmic method, modelling techniques and heuristic problem-solving techniques.
- To help students develop the ability to analyse, perceive structure and establish structural relationships.
- To help students develop written and verbal precision of argumentation and

speech.

- To develop students' ability to work with text and other mathematical materials and make them independent learners. (Schoenfeld, 1990)

2.1. Education

The topic of the dissertation has been studied over the years by researchers in the field. They have described various aspects of education for the development of mathematical thinking. For the purposes of the dissertation, the approach to the questions and hypotheses posed is followed by a brief historical overview.

2.2. The new educational paradigm

The credo of the new educational paradigm for **lifelong learning** is embedded in the hypotheses of the thesis – for students (1) and teachers (6). The development of technology, artificial intelligence and globalisation are some of the factors for the transition. The task of the research is to compare publications and normative documents that reflect the transition to the new educational paradigm.

2.3. Motivation

Intellectual curiosity and emotional engagement affect motivation. D. Poia's statement is still relevant even today – "the best motivation is the student's interest in his task." (Pólya, Mathematical Discovery, 1981)

In her dissertation N. Nikolova derives and proves the hypothesis: "The synergy between project and research approaches contributes to the full assimilation of educational content in informatics and information technologies, at the same time building key competences and important non-technical skills among students, increasing motivation for learning and creating a basis for sustainability of the achieved results". (Николова, 2016) The proximity of the scientific fields gives reason to expect similar results and conclusions when testing the 1. Hypothesis – increase in motivation and sustainability of acquired knowledge when working with applied tasks.

2.4. Creative thinking in mathematics

One of the 21st century skills is creativity, and its stimulation through regular work on applied tasks is set out in the second hypothesis. When solving textual problems, creativity is expressed in the approach to creating the "scaffolding", in the combination of methods used, in the application of digital technologies both in the solution process and in the documentation of its presentation. In the analytical tasks, the student is in the role of discoverer of one or more approaches to deal with the problem situation. An additional stimulus for creativity is provided by synthesis tasks, where the student is in the role of a creator.

3. CHAPTER: CONTEXT OF THE PROBLEM

Applied tasks are included in the mathematics course from the first grades. Before they can read, children learn to pick up and take out, for example, candy. The tasks engage their imagination, but also relate to familiar objects and their actions with them.

3.1. Paradox – education and expectations

Parents emphasise the importance of mathematics for their children's education and want them to be taught in mathematical schools, expecting better education, and learning in a competitive environment. Unfortunately, the scores rather than the knowledge remain the main objective. Mathematics, the Queen of all sciences, is becoming a shepherd's pole for school admissions.

The present paper deals with issues related to the place and role of applied tasks in the mathematics course and their importance for the development of 21st century skills, cognitive and metacognitive competences, and functional literacy of learners. It is our duty as teachers and parents to raise average scores.

For the purposes of the dissertation, different forms and tools of supplementary education are considered, as well as their role and importance:

- extracurricular activities,
- educational platforms,
- educational videos,
- forums,
- online tests and materials.

The nature of the tasks is a suitable stimulus for increasing intellectual curiosity and interest in young people. In the article Problem solving model in mathematics, it is stated that one of the signs for classifying mathematical problems is according to the "number of unknown components involved". (Desislava Georgieva, 2018). It was noted that applied tasks are rare, which underlines the need to supplement the mathematics course with such tasks on real-life topics. The same problem is discussed by Philip Petrov in his article "Application of the research approach in informatics education through a popular mathematical focus (A new look at old ideas)". His research shows that current Bulgarian textbooks have too many mathematical problems and very few have direct practical application. "It is a motivated opinion that it is necessary to stimulate to a greater extent a research approach in the teaching of informatics with the help of practical tasks with foci, games and sophisms, and the introduction to the topics should be carried out by placing students in a problem situation in order to causing stronger cognitive dissonance.“ (Petrov, 2022) The conclusion drawn cannot be directly applied to mathematics teaching, where prior knowledge is required to include such 'challenging' tasks. However, it is undeniable that placing students in a problem situation when working on the topics is a stimulus for cognitive processes.

In his book "The Mathematical Discovery", (Pólya, Mathematical Discovery, 1981)

distinguishes four types of grading problems:

(1) **One rule under your nose**: The problem can be solved by straightforward mechanical application of a rule or by straightforward mechanical imitation of an example. Moreover, the rule to apply or the example to follow is thrust under the nose of the student; typically, the teacher proposes such problems at the end of the hour in which he has presented the rule or the procedure. A problem of this type offers practice but nothing else; it may teach the student to use that particular rule or procedure, but has little chance to teach him anything else. And there is the danger that, even of that single rule, the student will acquire just "mechanical," and not "insightful," knowledge.

(2) **Application with some choice**: The problem can still be solved by the ap¬ plication of a rule learned in class or the imitation of an example shown in class, yet it is not so immediately obvious which rule or example should be used; the student needs some mastery of the material covered in the last weeks and some judgement to find the usable item in a certain limited region of search.

(3) **Choice of a combination**: To solve the problem, the student must combine two or more rules or examples shown in class. The problem need not be too difficult if a somewhat similar (but not the same!) combination has been discussed in class. Of course, if the combination is quite novel, or if many pieces of knowledge must be combined, or pieces of knowledge from chapters wide apart, the problem may demand a higher degree of independence and may become quite difficult.

(4) **Approaching research level**: It is scarcely possible to draw a sharp line of demarcation between the kind of problem we have just considered in (3)

As the complexity increases, so does the educational value from the point of view of the goal – development of mathematical thinking.

For the purposes of the dissertation, the types of tasks in the textbooks at the different levels of education will be studied. Of interest is the quantity and the variety of those used in parallel with those in foreign countries' educational systems. The study is divided into a review with sample tasks from the junior high school course (5th – 7th grade), the first gymnasium stage (8th – 10th grade) and the second gymnasium stage (11th – 12th grade). A parallel has been drawn with the choices made according to the spiral model of education.

One of the examples considered is how the above-mentioned exercises can be supplemented with an applied task and moved from type (1) of D. Polya's scale to type (3).

Problem 4: A ball is fired from the second floor of a building. Its trajectory is described by the function $h(t) = -t^2 + 2t + 3$, where t represents the seconds since launch and $h(t)$ is the height in meters above the ground. Determine:

a) From a height of how many metres has the ball been thrown?
b) How long does it take for the ball reach the ground.

c) The interval for t in which it is meaningful to consider this function.
d) The time taken for the ball to reach its maximum height in meters and the height of the ball.
e) Visualize, through a suitable application, the trajectory that the fired ball describes.

Item breakdown by difficulty level: The questions are graded according to the difficulty structure described. The first two subsections – a) and b) – refer to the initial and final position – difficulty (1). The searched interval corresponds to the valid values – difficulty (2), the determination of the critical point, which is the vertex of the parabola, corresponds to difficulty (3) – interpretation of the special points of the graph of the function, and the last sub-item is related to the digital literacy of the students, as it requires working with an application through which the process under consideration can be visualised.

Task Breakdown by Task Criteria: The task covers all the items in the task selection criteria list. The aim is an applied approach to the quadratic function. Knowledge is combined to find points, which implies different approaches. Here the teacher's role is, after following the students' decisions, to negotiate and compare the studied ways of finding the special points. It is appropriate to give students the opportunity to justify the strategy they have chosen. Feedback on the learning outcomes is given according to the students' approach to the problem.

Questions for students to ask themselves: **What** exactly am I doing? (Can I describe it precisely?) **Why** am I doing it? (How does it fit into the solution?) **How** will it help me? (What will I do with the result when I have it?) (Schoenfeld, 1992)

The dissertation describes in detail the solution, typical mistakes and how the students, under the guidance of the teacher, can deal with the problems.

3.2. Matura and admission to universities

The reduced core material in mathematics is a stumbling block for admission to foreign universities. The addition of normative documents with the possibility for students who did not study mathematics as a profiling subject to take the matura in mathematics is a relief, but it does not solve the problem of the discrepancy in the mathematical preparation of Bulgarian students compared to their peers from other European countries. The task of this dissertation is a comparison with the normative documents in support of the 5th hypothesis. According to the strategic framework, which is based on the previous recommendations of the European Commission for the development of education in our country and the current documents, the aim is to "activate exchanges with teachers, staff and students in foreign educational institutions and inclusion in international networks" (Стратегически документи, 2021). The limited set of applied tasks is a component that should be developed in terms of problem solving, 21st century skills and lifelong learning to make our graduates competitive and mobile.

4. CHAPTER: INTERNATIONAL ASPECT, ANALYSIS, PARALLELS

For the purposes of the dissertation, a parallel is made between the Bulgarian educational system and the Russian, German, Austrian, English, Italian and American. The review makes a comparative analysis of the role, types and number of applied tasks represented in a high school course in other countries – Austria, Germany, England, Italy and in Bulgaria. The purpose of the review is to analyse where our education stands in the European context. We are looking for ways in which, by enriching the course of study with such tasks, we can improve the results in mathematics and the motivation of students, how we can better develop mathematical thinking, competences, lifelong learning skills and 21st century skills.

The review is again divided into tasks from the junior high school course and the first and second years of high school.

One of the tasks under review is:

11. We have 200 metres of wire with which to fence a rectangular yard, one side of which is the continuation of a wall. Find the dimensions so that the enclosed area is the largest.

Analysis: In reality, this task also boils down to compiling a quadratic function and determining its maximum. It is interesting how the solution runs in parallel with the competence framework:

- analytical reading – what is given, what is sought, separation of known and unknown quantities,
- critical thinking – what is the meaning of the wall,
- creativity – sketch of the problem,
- problem-solving – what is the relationship between known and unknown quantities,
- knowledge of mathematics – how to find the circumference and face of a rectangle, how to make a model with one variable, how to find the largest value, what is its meaning,
- assessment – what are the limitations, how are the values found interpreted,
- self-assessment – are the obtained values meaningful, are there other ways to solve the task.

The proposed solution is compared with possible ones generated with ChatGPT.

The analysis of foreign educational systems and the problems considered in them provides a guideline for enriching the set of activities by

- Graphical pattern analysis – "recognition" of features tasks
- Different approaches to the analysis of a function, depending on the type of the provided points – e.g. zeros or vertex.
- Early-stage solutions to extreme tasks.

The fourth chapter traces the combination of digital technologies in solving applied

tasks. Today's students are Generation Z, and teachers being trained now will be working with Generation Alpha students (Eldridge, 2023). These are teenagers who spend hours in front of screens, who are comfortable with digital technologies and see them as a natural part of their lives. In our dynamic modern times and following the paradigm of lifelong learning, it is inevitable that teachers and future teachers need to be up to date, informed and use technology critically and appropriately in the educational process – 6. Hypothesis. From the meta-analysis by A. Chung and R. Slavin on the effectiveness of educational technologies in improving mathematics achievement in schools, the conclusion is clear: the modern world is digitising, and schools cannot be an exception. According to the researchers, the question is not whether, but how teachers can successfully use the new tools for educational purposes (Cheung & Slavin, 2013). The authors emphasise that technology can be an addition to, but not a substitute for, education.

Ev. Sendova emphasizes "To instill in our students the idea that mastering technology is important, but the most important thing is to use it NOW so that we learn to deal with complex problem situations in the FUTURE " (Сендова, 2014)

According to the programs of the Ministry of Education and Culture, in some methodological units work with appropriate software is planned, but we cannot limit ourselves to this. The characteristics of digital applications provide opportunities for closer integration with the curriculum in almost all topics studied in the mathematics course. One such implementation is the VirMathLab (ИМИ–БАН, 2013), which is a leading example of sharing experience and materials. A collection of GeoGebra-developed exercises and tasks is freely available. Toni Chehlarova's guide "Development of a system of practical-applied tasks for the formation of key competences in mathematics" aims, in examples, to propose a change in the learning process, consistent with the new reality (Чехларова, 2014). The team advocates the idea that "... mathematical modelling and applications of mathematics are not represented sufficiently in the school course" and all models in such tasks are simplified and do not correspond to reality (Кендеров & Чехларова, 2021). The use of appropriate software when solving the problems makes it possible to expand the range of applied tasks that correspond in complexity to real life. Such tasks, in turn, increase the motivation of students, who see how mathematics has a real practical implementation – 1. Hypothesis.

4.1. Math Essay and the Internal Assessments

A detailed analysis of the mathematical essay as a tool in mathematics education and its added value has been made. A part of the aspects, importance and the relation are referred in Table 3.

Table 3: Competencies and criteria

Competence	**Evaluation criteria**
Mathematical literacy	Difficulty level of the problem, correctness of the solution

Competence	Evaluation criteria
In-depth knowledge of the subject	Mathematical reasoning
Digital literacy	Formatting text and presentation, handling digital tools for visualizing graphs and formulas.
Research approach and innovation	Choosing a topic, finding a problem that is not unexamined in the lessons and in the textbook
Heuristic thinking	Searching for a solution to an unsolved problem
Critical thinking	To the information in the Internet space and to the reasoning and results it realizes
Self-initiative	Freedom of choice
Self-direction	Systematic and motivated work, qantity and quality requirements cannot be met overnight
Self-assessment	With clearly defined criteria, setting objective self-assessment, ability to see the work from the side
Responsibility	Choosing a topic; meeting deadlines; citation

The mathematics essay teaches you how to present ideas, formulate a problem and communicate its solution, not just with formulae and calculations, but with clear explanations in language that a wider audience can understand. In this way, you will go beyond writing homework and demonstrate a deeper understanding of the nature of mathematics. Students are expected to have experience of writing essays in other fields such as literature, history and philosophy. They need to transfer their skills in creating and structuring the task from these areas and combine them with mathematical formulae and judgements. In a mathematical essay it is important to be precise and to express ideas clearly. The teacher's role is to prepare learners for this next step in their development, both at the level of mathematical thinking and at the level of specific competences. The transfer of skills between subjects is the glue that holds education together.

4.2. Applied tasks according to Bloom's taxonomy

In view of the issues of the dissertation, a section of the applied tasks was made (Fig. 1), which follows the revised Bloom's taxonomy according to Anderson and Krathwohl (Anderson & Krathwohl, 2001).

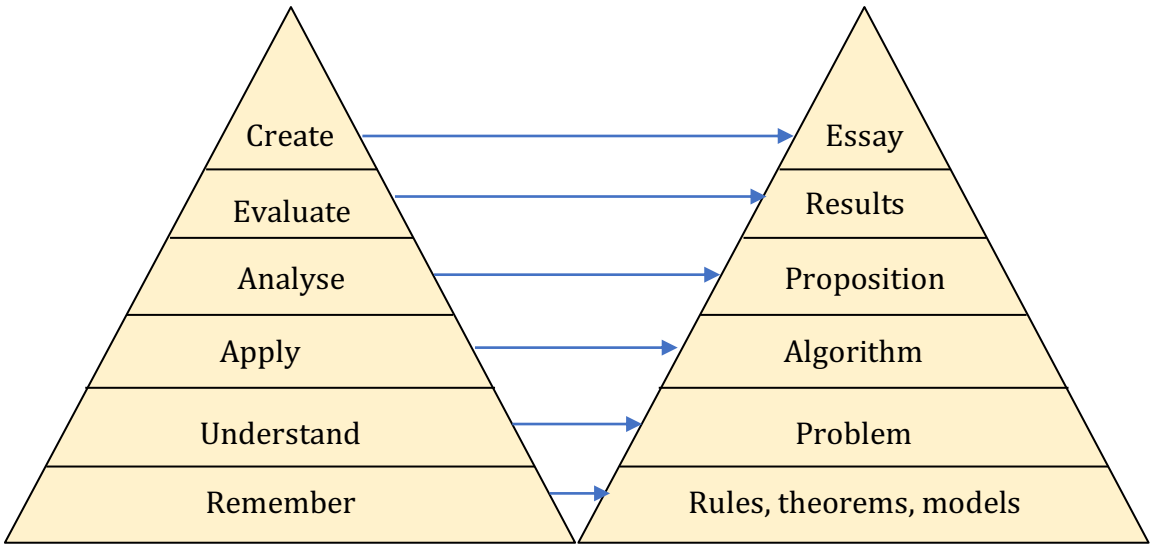

Fig. 1: Revised Bloom's Taxonomy according to Anderson and Krathwohl in parallel with applied tasks

A research axis has been added to each level. It aims to illustrate how applied tasks link cognitive levels and 21st century skills. For this purpose, both specific examples from the mathematical content of the course and competences corresponding to the level are specified.

Remember *the surface formula, the method* of finding the largest and smallest value of a function, *Cauchy's inequality*. It is expected that the planned mathematical knowledge is mastered – mathematical literacy.

Understands the individual words from the context of the task, the text as a whole and the *problem posed* – what is being required in the task. In analytical reading it is important to cover the whole condition – reading literacy, handling the information provided, discipline, concentration, problem solving. **Understands** how to apply learned theoretical material to a specific task – mathematical literacy, problem solving.

Applies and combines memorised formulae, learned *algorithms* or accumulated experience to solve such tasks – mathematical literacy, combinative and critical thinking, *problem solving*, independence, self-direction.

Analyses the meaning of the *content* of the task and the meaning of the values obtained in the context of the given problem – critical thinking, management of information provided.

Evaluates the strategy chosen, alternative approaches, which *results* obtained are meaningful according to the task, formulates a logical answer – critical thinking, evaluation, self-direction, and self-evaluation.

Creates additional conditions or changes to the task under consideration, a new task, a mathematical essay, a presentation on a given topic – creativity, mathematical competence, problem solving, self-direction and self-evaluation. The mathematical essay is placed squarely in the final stage – **creating**, which is defined as "bringing together elements to form an intelligible or working whole; reorganising elements into a new pattern or structure by generating, planning, producing" (Anderson L. , et al., 2000).

5. CHAPTER: FIELD RESEARCH

The conducted independent research was preceded by observations of graduates from the German School Sofia (DSS) on of the German schools abroad (DAS schools). Most of the students are Bulgarian.

The pre-observed case-study and the parallel with the expected competences according to the program of the Ministry of Education and Science are the basis of the empirical information and serve as an initial orientation in the problem (Кожухарова, Ганчев, & Делчев, 2004). Following the method of qualitative pedagogical research based on participant observation, an understanding of the problem faced by the researched individuals is reached. The collected information makes it possible to set the research tasks of the present dissertation and formulate the 3rd hypothesis – theory based on data (Glaser & Strauss, 1967). The hypotheses of the present dissertation consider, on the one hand, advanced reading comprehension and functional literacy of students – working with text and image. Another axis of consideration is the competences and the 21st century skills.

5.1. Preparation – selection of specific tasks

Two different types of tasks are used for the purposes of the study. Their specific representatives are radically different in their formulation. One (problem A) clearly defines the characteristic points of a function whose coefficients must be determined. The aim is to assess the level of knowledge of the relevant mathematical material. The second (task B) consists of a descriptive text and an illustrative graph. In this case, based on an analytical reading in combination with a reading of the graph, the conditions must be separated to find the same coefficients. The constraints are the same in both tasks, the mathematical model is the same and so are the final answers.

In Fig. 2, the two tasks are presented in parallel. Both are available online in Internet and accessible via browser. The technical implementations is via Jotform platform (Jotform, 2022).

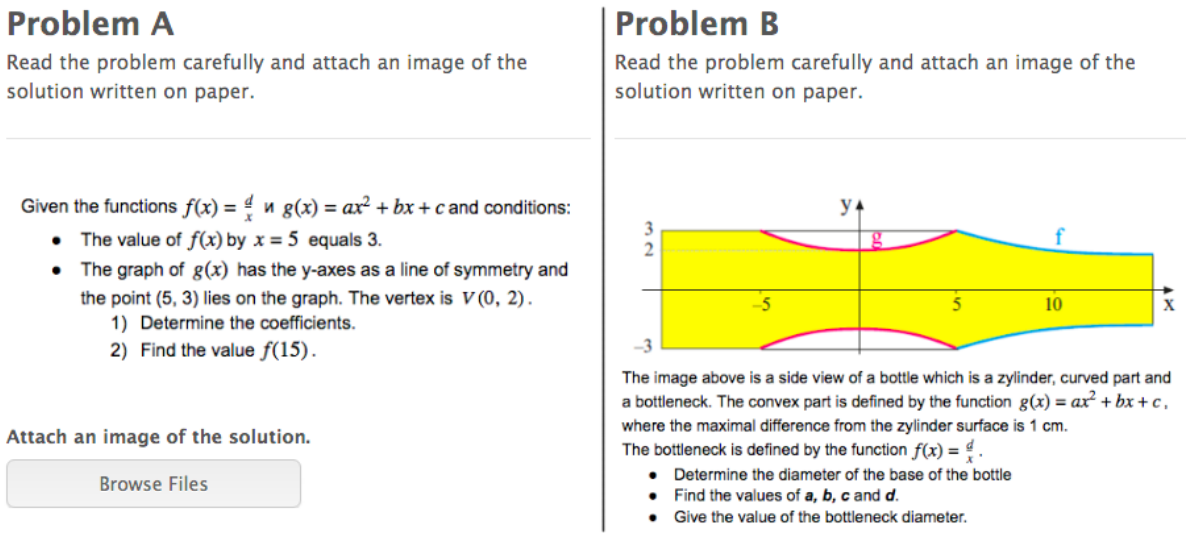

Fig. 2: Formulation of tasks – research Phase '21

As a basis, a task from an active mathematics textbook for the 11th grade in Germany (Oberstufe, 2017) was used. The task was translated, adapted and corrected after the preliminary research. The conditions of the two tasks of the second field study followed the pattern of the first study.

5.2. Methods and tools

To verify the hypotheses of the dissertation, a theoretical-empirical (applied) study was carried out, which by its nature is diagnostic and verification – to analyse the difficulties in solving a textual problem with an image and to draw a parallel with the solutions of the purely mathematical model. Experimentation according to the conditions is natural; according to the conduct – main; a high "purity" of realisation was observed.

The research tools include two representative tasks and a survey. The didactic test is individual; criterion; for continuous control of performance and attitude. The expected response is open-ended – a detailed and reasoned solution to the task with clearly stated values of the quantities in question.

The survey part is a questionnaire (written, absentee) with closed answers, which are of several types:

- Open answer,
- yes/no
- scaled
- menu responses with a complete list of cases.

5.3. Evaluation criteria and indicators

In order to evaluate the submitted solutions qualitatively, a multivariate model of the expected solution was created for both tasks, so that they could be evaluated at different levels. The written work submitted by the students is thoroughly examined and

coded according to the success rate. The indicators defined in the dissertation are decomposed according to the specificities of each task and the research methods chosen.

1. Degree of assimilation of knowledge and skills
1.1. Specifying one or more of the search parameters
1.2. Determining functional value
2. Solving depth
2.1. Analysing individual characteristics – parity, symmetry
2.2. Top of the parabola
2.3. Special points
3. Thoughtfulness of the problem
3.1. What is the purpose of the task?
3.2. What are the meaningful values of the parameters and the variable?
4. Transfer of knowledge
5. Observational skills when working with text and images.
6. Independence
6.1. Duplication of submitted works.
6.2. Searching for a ready-made solution on the Internet
7. Creativity and initiative
7.1. More than one solution.
7.2. Comment on the condition
8. Discipline and responsibility
8.1. Quality work, although not to be graded.
8.2. Effort (Draft – White)
9. Self-criticism
9.1. Parallel between self-assessment and quality of delivered decision.
9.2. Parallel between difficulty rating and actual coping
10. Digital skills
10.1. Type and format of transmitted files (for the online experiment)
11. Critical thinking
11.1. Is the sketch correct or just for reference.
11.2. Is the task pre-defined?
11.3. Which resulting values make sense in the given context.
12. A clearly worded answer.

2) The vertex of the graph of the function $g(x) = ax^2 + bx + c$ is $V(0, 2)$.

$$\Rightarrow g(0) = 2$$
$$g(0) = c$$
$$\Rightarrow c = 2$$

Indicators 1.1, 1, 2, 2.2

The graph is shifted 2 units up visible by the form:

$$g(x) = ax^2 + 2$$
$$\Rightarrow b = 0$$

Indicators 2.1, 1.1

The information about the line of symmetry is duplicated by coordinates of the vertex, which gives the equation of the line of symmetry:

$$y = y_v$$
$$y = 0$$

Indicators 2.1, 5, 7.2

Another way to determine the value $b = 0$ is using the fact that the function is even $g(-x) = g(x)$.

$$\left|\begin{array}{l} g(5) = 3 \\ g(-5) = 3 \end{array}\right. \qquad \left|\begin{array}{l} 25a + 5b + 2 = 3 \\ 25a - 5b + 2 = 3 \end{array}\right.$$

$$\Rightarrow b = 0$$

Indicators 2.1, 7, 11

Using the coordinates of the point lying on the graph (5, 3)

$$\Rightarrow g(5) = 3$$
$$25a + 2 = 3$$
$$\Rightarrow a = 0{,}04$$

Indicators 5, 1.1, 1.2

The final values of the parameters are $a = 0{,}04$; $b = 0$; $c = 2$. The observed function is $g(x) = 0{,}04x^2 + 2$.

Indicators 12

5.4. Research phase

According to the topic and objectives of the dissertation, a review of literature and research in the field of the importance of mathematics for the development of cognitive skills and the new educational paradigm of lifelong learning was carried out. The expected competences according to the plans of the Ministry of Education and Culture and EC documents, foreign and Bulgarian textbooks, study aids and used examination materials confirm the relevance of the hypotheses. Similar studies have been carried out, with the frequently discussed problem being the excessive use of digital technologies. The observation that precedes the research is consistent with the third hypothesis and provides the basis for preparing and conducting a field experiment.

6. CHAPTER: ANALYSIS OF THE RESULTS

Preliminary research is based on participatory observation. The data collected allows us to formulate a 3rd hypothesis. Hypothesis. The pre-observed group is small and unrepresentative of the Bulgarian educational system; therefore, it is not possible to

present a theory based on facts, but only a hypothesis. The comparative analysis of the teaching materials in the Bulgarian and foreign education systems points to the insufficient use of applied tasks in the secondary school curriculum in our country, which supports the formulated 3rd hypothesis. Due to the restrictions imposed by the pandemic, the studies in this dissertation do not continue the method of qualitative pedagogical research but develop according to the rules of empirical pedagogical follow-up. The topic of the dissertation is the role of applied tasks in the development of key competences and 21st century skills. Commenting on the observed qualities of the students is a reason for sticking to empirical analysis at the level of description. (Бижков & Краевски, 2007, p. 232).

6.1. Evaluation of the solutions from the two experiments

Descriptive analysis focuses on the quality of decisions, in parallel with the skills and competencies under consideration. Performance is quantified and the results for both studies are shown side by side for comparative analysis. The performance of the control and focus groups in the first study is close, which is in line with what would be expected of high achievers in mathematics. However, even well-educated students have more difficulty with the non-standard task.

6.2. Summary of Phase '21 results.

A total of 151 responses were collected, but after filtering out duplicate records, 138 unique survey completions and assignment submissions were considered.

According to 3rd hypothesis, the success rate in solving the two tasks should be significantly different. Due to the specificity of FG and CG, the hypothesis is not confirmed, but this is not a reason to reject it. Observations of the results show that, despite the high mathematical preparation of the participants, there are differences in the success rate, feedback, and self-evaluation. On an example work, the indicators that are included in the research methodology are plotted (Fig. 3).

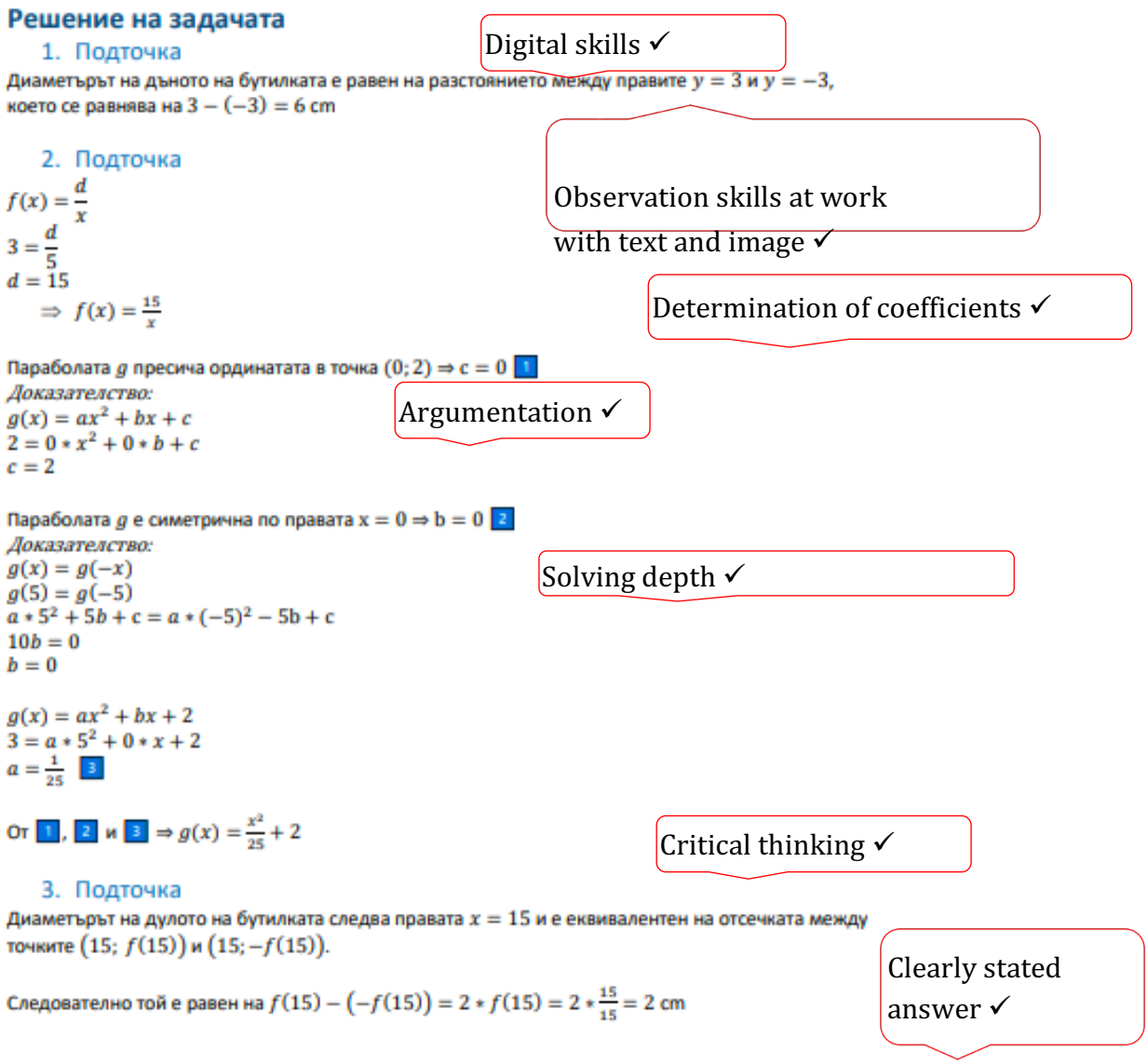

Fig. 3: Solution to Task B

The habit of strictly applying formulae and following algorithms to solve problems limits creativity and a rational approach to solving problems. It is interesting to note that no student suggested more than one solution to the problem, even though many found the task easy and had enough time. There is a lack of innovation and activity that can occur with alternative solutions. Textbooks often offer several solutions to a problem and many teachers also encourage different approaches to the same problem. Unfortunately, despite having unlimited time to solve the problem, no student added a second way of solving the problem. The only observable expression of innovation is the choice of a tool to implement the recording of the decision – working with an application on a digital device.

Task B has a direct impact on 21st century skills – creativity, problem solving, ability to manage the information provided. According to hypothesis 2, applied tasks develop these skills. The research results indicate that there is limited experience of working with such tasks.

Based on the processed written work and the data from the survey, the following was done: a comparison between self-assessment and assessment; lead time; analysis of the submitted files from a technical point of view; feedback analysis.

A similar analysis was carried out on the data collected in phase '22.

6.3. Discussion

The criteria and indicators considered are:

- **Mathematical competence** – ability to read the requirements of the given task,
- **Mathematical knowledge** – given the profile of the participants, high results are expected,
- **Research approach and innovation** – there is no proposal with more than one solution,
- **Critical thinking** – no one noticed the over-determination of the task and only one student noticed the discrepancy in the written scale,
- **Self-initiative** – only one student suggests a more appropriate word for opening the bottle,
- **Self-direction** – most students followed strictly the application of formulas, without any analytical parallel between text and image, the formulation of an answer, the overall layout of the solution,
- **Responsibility** – attitude to presentation – most of the work is in the form of a draft,
- **Creativity and presentation skills** – mainly expressed in the layout of the decision and the choice of tools,
- **Digital skills** – technical implementation, layout and files submitted. This is an additional observation obtained due to the exceptional conditions under which the experiment was conducted, but it provides an insight into the digital culture of the participants. An interdisciplinary link with information technologies is established without any preconceived notions.

The detailed breakdown of the solutions according to the established criteria and indicators confirms the importance of the applied tasks for the development of competences and 21st century skills – creativity, problem solving, ability to handle the provided information.

7. CHAPTER: LOOK BACK

After the definition of the problem, a thorough review of the literature and an analysis of the research carried out according to the method of D. Polya, it is time for a wider review.

Bulgaria is a country with traditions in mathematics education. The national mathematics team and individual students have been winning awards for years by

participating in international mathematics competitions. In addition, the country's PISA results are low and unsatisfactory. This points to educational problems related to the development of students' skills for an analytical, creative and combinatorial approach when dealing with a variety of tasks. The aim of the study is to analyse students' performance in solving similar mathematical problems. In addition to the high academic level, the dynamic times in which we live require that the preparation in mathematics is not reduced only to the application of algorithms and formulas. Skills are needed for in-depth analysis, interpretation of data, synthesis of information in order to build a model through which a solution to the problem can be achieved. Then, in retrospect, it is necessary to communicate the decision taken clearly and rationally.

7.1. Problems for 11. – 12. degree student

Application problems relate algorithm steps and function critical points to their meaning in different contexts. Tasks related to real problems are the natural extension of the theory and the missing applied side that has existed for years in foreign educational programs.

7.2. Problem 1

Problem: *Discuss the function:* $f(x) = (x-1)(x-2)^2$ (Иван Тонов, 2021).

We can safely put this task in the exercise column. "We have a goal and the path to it is clearly defined" (Тонов, 2012). When teaching function study, an algorithm is specified. The study of this polynomial function has a clear structure. No heuristics are needed to aid in the decision.

"When the task has an interesting and intriguing formulation, and especially when it allows for an unexpected solution, working on it is more attractive and motivating." (Тонов, 2012)

7.3. Problem 2 – Applied

After injecting experimental mice with a mild drug, the levels of eosinophil* leukocytes in their blood (*indicators of an allergic reaction) are monitored. We assume that at the beginning the level of these enzymes was close to 0%. The level (1 m unit = 10%) of the drug in the blood during the period of action is described by the function $f(t) = t(t-2)^2$, and time is measured in 24 hours.

1. How long does it take for the blood to completely clear?

2. For which interval of the variable does the considered function describe the stated problem?

3. When are the highest values of leukocytes reached and what are they in percentage?

4. When are levels dropping the fastest?

5. If immediately after the moment of greatest decline, the change in enzyme level

can be described by a linear function, then what would its equation (graph) be? How will this change the moment of complete blood clearance?

6. How long does the allergic reaction last (reference values 0-6%)?

Several mathematical solutions are proposed in the dissertation, accompanied by a description of the role of the teacher and the importance of the individual parts for the development of the competences and the 21st century skills. One of them is using software with the emphasis on the interpretation of the results, as an example of this are the following figures.

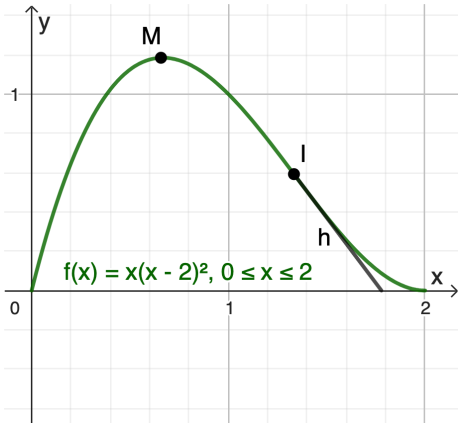

Fig. 4: Part of the tangent line at I

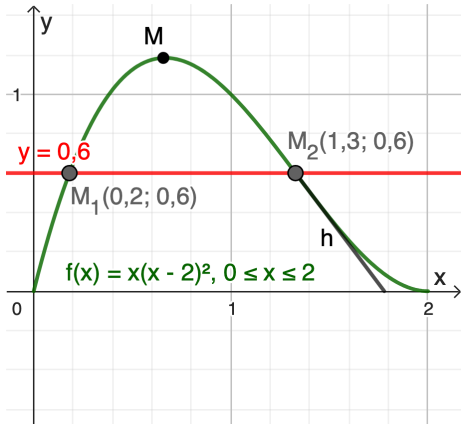

Fig. 5: Intersections of f(x) and y = 0,6

"A student's mathematical experience should not be considered complete if he has never had the opportunity to solve a problem of his own making." (Pólya, How to Solve It , 1945) As a next step for independent work, or for homework or as a project, the following task can be given:

7.4. Problem 3 – Creativity

Problem: Given the function $f(x) = x(x-2)^2$

Write a problem in which the given function describes a process chosen from the fields of geography, biology, chemistry, physics, technology, and entrepreneurship. Ask process-relevant questions related to function values, extrema, inflection point, examine tangents to the graph of the function.

In class, the teacher can stimulate the description of a problem from another field of knowledge. An example wording is proposed in the dissertation.

Teachers and university lecturers are faced with the complaint of learners that an exam paper includes "unpractised material". The complaint "We didn't do that in class" is quite inappropriate for a mathematics subject where real mastery of the material is demonstrated by combining knowledge and skills. A suitable examination regulation can eliminate such dissatisfaction among learners and would encourage them to seek suitable examples of exercises.

DIRECTIONS FOR FURTHER DEVELOPMENT AND CONCLUSIONS

Future work on the theme

This dissertation examines the role of applied tasks in mathematics education. According to the overall research, their pedagogical potential is currently underdeveloped.

The proposed 3rd hypothesis has not been categorically confirmed by the research conducted, but there is no reason to reject it. A natural extension is the planning and implementation of representative research. The task used should be specified. The accompanying questionnaire should be further developed in consultation with statisticians. It is necessary for the field research to be under direct observation, enriched by additional interviews and discussions with both students and teachers. The analysis of the results provides an opportunity to assess the current situation.

Development Guidelines

- **Research among students** – to achieve representativeness of the data, the group of participants should be expanded with representatives from all over the country, high schools of different profiles and specializations, first and second high school stage.
- **Research among current and future teachers** – to track the attitudes and experiences of teachers (master's degree students, teachers attending additional or post-graduate courses) such research should be carried out in two formats. One should be identical to that of the students. A source of more conclusions would be a "qualitative experiment" in search of effective minimal intervention approaches in guiding students to find and reason for their own decision.
- **Analysis of the results** – detailed quantitative and statistical analysis of the data from the conducted representative study. Based on the obtained results, make appropriate assessments and conclusions. On their basis, grounds for proposals for supplementing the school mathematics course with the relevant set of tasks should be formulated.
- **The applied tasks in the past years** – invaluable experience can be gleaned from old Bulgarian textbooks. For this purpose, a chronological tracking of the amount and types of applied tasks from the beginning of the last century to today is necessary. Tasks can be described and made available in a shared repository. Their thematic structuring makes it possible to abstract from the annual distribution according to the current program of the Ministry of Education and Culture. Experimental curricula should also be considered, their content goes beyond the set framework in search of new approaches and offers non-standard tasks that deserve attention. In them, you can find many interesting, tried-and-tested tasks that hide currently missed opportunities.
- **Training material** – formulation and approval of a scheme of criteria for assessing the complexity of an applied task, tools for creating applied tasks.

- **Integration** with STEM education. Mathematics for mathematics through appropriate tasks e.g. linear programming problems.

Conclusion

Execution of the research tasks

The development of the dissertation follows the set tasks of analysing the role of applied tasks in mathematics education. A review of the literature and the results of related research in the field will be carried out. The current regulations in our country and those of the European Commission for the development of education are compared regarding the expected skills and competences. Selected topics from current textbooks in the country and those from established foreign systems are compared to assess the current level of use of modelling tasks at the upper secondary level. The role of applied tasks as a tool for making cross-curricular connections is analysed. A scenario of applied tasks in terms of structure and content is developed. The axes of consideration of applied tasks are described. The process of applied problem solving according to the identified key competences and 21st century skills is outlined. Their importance for the development of students' cognitive and metacognitive skills is analysed.

Validation of the hypotheses

The reviewed literature sources, normative documents and programmes, similar studies in the field confirm the 1. The hypothesis that applied tasks increase interest and motivation and develop lifelong learning skills and sustainable knowledge.

The content and process analysis of applied problem solving, which describes a problem close to real life, is refracted through the prism of the competence model and 21st century skills. On the basis of a literature review and specific examples, 2. the hypothesis that such tasks help to achieve the goals of mathematics education by developing to a greater extent the identified key competences – critical thinking, analytical reading, problem solving, creativity. The role of these tasks in the development of analytical-euristic thinking and presentation skills in the formulation of an argumentative statement is discussed. Both criteria described in the literature and those created by the authors are used to illustrate how the competences are 'brought to life' in practice through the use of applied tasks. The analysis of some of the results of the pedagogical research conducted also indicates support for hypothesis 2.

The results of the field-based pedagogical research are not sufficiently conclusive to confirm the third hypothesis. That applied tasks are more difficult even for students with a good mathematical background. However, they are not a reason for rejecting the hypothesis, but provide a basis for preparing and conducting a representative study, because the literature review of similar studies confirms the hypothesis.

The influence of the applied tasks on concentration, cognitive and metacognitive patterns is indicated. Based on the literature review and analysis of additional data from the pedagogical study, 4. the hypothesis that purposeful work on such tasks is important for building cognitive and metacognitive patterns, trains concentrated reading, and

increases self-discipline. Combined psychological and pedagogical research should be conducted to achieve better measurability.

As for the specific tasks, an example is given, such as the transition from the result of a real problem to a mathematical model for the implementation of "practical" knowledge, as defined in the strategic documents of the Ministry of Education and Culture. The problem of insufficient applied tasks in mathematics in secondary education is presented in an argumentative way. It has been considered that, despite regular changes in the curricula, the costs of preparation do not meet the requirements of a European education. The conclusions drawn confirm the need to supplement the school course with applied tasks to increase the competitiveness and mobility of high school students, thus confirming the 5th hypothesis.

Another axis of comments is the new educational paradigm for lifelong learning, where the leader is a personal example for teachers – informed and applying current approaches in education, using digital technologies, interested in the materials and applications available to students, able to evaluate their effectiveness. The reviewed documents, the analysed process of solving applied tasks and the place of educational digital technologies in education confirm the 6. The hypothesis that teachers should also develop their skills, learn, try and get to know new technologies and tools.

It is clear from the presentation that the creation of quality applied tasks should be a joint work of mathematics teachers with other specialists. This process naturally creates cross-curricular links and visualises the transfer of knowledge embedded in the course. Pupils' creativity should be seen as an additional source of inspiration in this activity. Integration with STEM education and applied tasks from mathematics for mathematics provide direction for the future development of the subject.

CONTRIBUTIONS

Scientific contributions

1. A structured review of literature has been carried out, analysing the role of applied tasks in mathematics education, describing their importance for the development of cognitive and metacognitive skills, the key competences and the 21st century skills.
2. A comparison was made between the normative documents in force in Bulgaria, including the programmes of the Ministry of Education and Culture for the mathematics course, and the documents of the EC for the development of education in the context of applied tasks.
3. Analytical is the place of applied tasks in the combination of classical and innovative didactic approaches, including digital technologies.
4. The data from the field studies carried out have been collected, coded and analysed, which can serve as a starting point for further studies in this field.

Scientific and Applied Contributions

5. A comparative analysis has been made of the variety and number of applied tasks in mathematics education in our country and in established foreign systems.
6. A structured review of alternative approaches such as the mathematical essay and the final project and their importance in stimulating students' creativity, self-direction, self-discipline and motivation will be prepared.
7. To propose a model for solving an applied task, based on literary sources and Poya's stages, with an additional breakdown of the individual steps according to the 21st century competences and skills addressed.
8. An analysis has been made of the formulation of the applied tasks from the point of view of the main parts of the structure, the setting of questions with an increasing level of difficulty, vocabulary, variety, correct content based on inter-subject links.
9. The role of the teacher in presenting, combining and upgrading knowledge is analysed in parallel with the modern educational paradigm for lifelong learning.
10. Materials and a plan for carrying out a pedagogical study have been prepared with the aim of assessing the ability to work with an applied task that has no analogue in current textbooks.

Applied Contributions

11. Criteria for evaluating an applied task are systematised.
12. An analysis of specific tasks studied in the mathematics course was made, following the spiral model.
13. Sample applied tasks have been developed to complement the mathematics course, accompanied by a solution that has been analysed along two axes – mathematical justification and analysis of the competences addressed.
14. Guidelines are outlined for the development of the set of applied tasks by building a question bank as a collective effort of teachers and students and by adapting good practices from established foreign educational systems.

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Declaration of Originality

I declare that the presented in connection with the procedure for obtaining the educational and scientific degree "Doctor" at Sofia University "St. Kliment Ohridski" dissertation on the topic: "The Role of Applied Problems from the School Mathematics Course for the Learning Purposes" is my own original work.

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